Problem Strings in Bridges

Problem Strings: A New Approach to Building Computational Fluency & Number Sense

by Martha Ruttle, Managing Editor

Recently a math coach wrote to me with a broad inquiry about problem strings. She wrote, "Problem strings are new to me and the teachers I work with. Can you give any guidance? Some do's and dont's?" I have to confess that problem strings are new to me, too! Pamela Harris and the other writers for Grades 3–5, all of whom are experienced teachers, introduced them to second edition Bridges. They have years of experience designing problem strings and using them in the classroom. I do not. So I consulted with some of the teachers and the math coach I've been working with, and I came up with the following guidance.

Early Challenges

All of the teachers I talked with shared that the first problem strings they did with students did not go well, or at least not as well as they'd hoped or as well as subsequent strings. Part of the challenge was that strings were new to them and to their students: they took some getting used to. Other teachers reported that initially it was hard to see what the big idea of the string was; they weren't totally clear about the connections they were meant to elicit from students. I hope that the Teachers Guide spells out the big ideas for later strings more clearly. We used a table format for the problem strings, because we felt it made it easier to follow the flow of problems and to spell out the connections among them.

Being Explicit with Students

It can really help to be explicit with students about what problem strings are and why you're doing them. One third grade teacher I spoke with found a string in her pocket the day she was introducing the first problem string of the year. Before launching the problem string, she asked students to talk about what a string is. Then she showed them the string in her pocket and slowly teased apart the threads of the string so that students could see how those threads, when intertwined, were stronger together. She explained that a problem string has mathematical threads that intertwine and together build stronger mathematical understandings. After the problem string was over, her students said, "Oh yeah, the problems go together like the threads!" This metaphor really made sense to them and they continue to refer to it when making mathematical connections during strings.

Helping Students Identify Connections

A fourth grade teacher shared that she also emphasizes the connections among problems by encouraging her students to regard the string as a sort of mystery. "What are the meaningful connections here? How does this all go together? What problem might come next?!" Building anticipation in this way is really engaging for her students, and it also focuses their attention on the heart of the problem string: the meaningful connections among the problems.

Some Dos and Don'ts

When implemented well and as designed, problem strings:

- Maintain a brisk pace and a tight focus
- Emphasize a specific computational strategy and/or a specific model
- Prompt students to use the connections between problems in the sequence to develop a specific strategy or gain a particular insight about an operation or about number relationships
- Over time, promote the development of a collection of efficient computational strategies

Do: Pose one problem at a time.

Don't: Pose or reveal all of the problems at once.

Do: Have students sit in an arrangement that allows you to walk around easily so that you can look at their work.

Don't: Have students cluster in a large group so that you cannot maneuver among them to see their individual work.

Do: Circulate around the room while students are working so that you can select students who have used the target strategies to share when you reconvene the class.

Don't: Call on students at random or call on students who have made errors. Discussing those errors might be productive, but the problem string is not the time for such a discussion.

Do: Keep the pace brisk.

Don't: Labor over each problem for too long or invite students to share a wide variety of approaches to solving a single problem. We do this frequently during other kinds of lessons, but problem strings are meant to be focused.

Do: Encourage students to record their work and then modify it to show the featured strategies.

Don't: Encourage students to continue using inefficient strategies if they can instead begin to adopt the more efficient strategies featured in the string.

Do: Follow the sequence of problems laid out in the problem string.

Don't: Improvise with other problems or modify the problems. They are meant to go together in a purposeful way, in the sequence laid out.

Do: Encourage students to draw connections between the problems in the string. Frequently pose questions like, "How can you use the problem you just solved to help solve this new problem?" or "Can you use what you just did to help solve this new problem?" These questions prompt students to draw connections from one problem to the next.

Don't: Allow students to approach the problems as isolated, unrelated computational exercises.

Do: Remember that the final problem is often not explicitly connected to the previous problems. Often, the final problem is not preceded by "helper" or preliminary problems. Instead, we hope that students will apply the strategy developed in solving the sequence of prior problems to solve the final problem independently.

Don't: Insist that students use the previous problem to solve the next problem if it doesn't make good sense to do so.

Bridges Implementation blog, September 30, 2014: http://bridges.mathlearningcenter.org/implementation/blog/problem-strings-new-approach-building-computational-fluency-number-sense

Problem Strings in Bridges 2nd Edition Grade 3

LOCATION	TITLE	MODELS
UNIT 1		
Module 3, Session 4	**Adding Tens** 28 + 10, 28 + 13, 28 + 23; 36 + 10, 36 + 16	Open Number Lines
Module 3, Session 5	**Get to a Friendly Number** 28 + 3, 28 + 7, 28 + 13…	Open Number Lines
UNIT 2		
Module 1, Session 2	**Groups of Stamps** 5 stamps per row @ 2¢ per stamp, 2 rows, how much money owed?	Arrays of stamps; each stamp in the array has the price on it.
Module 1, Session 4	**Stamps & Doubling** 2 × 4, 8 × 2, 4 × 2, 8 × 2, 4 × 4 × 4	Arrays of stamps; each stamp in the array has the price on it.
Module 2, Session 3	**Watertown's Window Washer** 3 × 5, 5 × 3, 4 × 4	Arrays of paned windows
Module 2, Session 4	**More Windows for Wally** 5 × 4, 4 × 4, 9 × 4	Arrays of paned windows
Module 2, Session 5	**The Watertown Post Office** 5 × 6, 6 × 6, 4 × 6, 8 × 6	Arrays of post office boxes
Module 3, Session 1	**Doubling** 2 × 4, 3 × 4, 6 × 4; 4 × 4, 8 × 4, 5 × 4, 10 × 4, 9 × 4	Open Number Lines
Module 3, Session 2	**Cats & Legs** 2 × 4, 4 × 4, 8 × 4, 10 × 4, 9 × 4	Ratio Tables, Equations
Module 3, Session 3	**Doubles & More** 2 × 6, 4 × 6, 8 × 6, 3 × 6…	Rectangular Arrays
UNIT 3		
Module 1, Session 3	**Give & Take** 36 + 99, 97 + 78, 299 + 647…	Open Number Lines, Vertical Combinations
Module 2, Session 3	**Constant Difference** 75 – 40, 74 – 39, 76 – 41, 77 – 42…	Open Number Lines
UNITS 4–6		
	No Problem Strings	N/A
UNIT 7		
Module 2, Session 1	**Partial Products** 5 × 3, 5 × 10, 5 × 13, 5 × 20, 5 × 23	Closed & Open Arrays, Equations
Module 2, Session 3	**More Partial Products** 6 × 10, 6 × 4, 6 × 14, 6 × 20, 6 × 30, 6 × 34, 6 × 54, 16 × 10, 16 × 4, 16 × 14	Open Arrays, Equations
Module 3, Session 1	**The Associative Property** 4 × 10, 4 × 8, 4 × 18, 4 × 80, 4 × 800, 6 × 9, 6 × 10, 6 × 19, 6 × 90, 6 × 900	Open Arrays, Equations
UNIT 8		
	No Problem Strings	N/A

Problem Strings in Number Corner 2nd Edition Grade 3

TITLE	MODELS
SEPTEMBER	
ADDING 2- AND 3-DIGIT NUMBERS **String 1** Using Doubles to Solve Near Doubles **String 2** Jumping by Friendly Numbers **String 3** Jumping to Get to a Friendly Number	Equations, Splitting Diagrams, Open Number Lines
OCTOBER	
SUBTRACTING 2- AND 3-DIGIT NUMBERS **String 4** Jumping by Tens **String 5** Friendly Tens **String 6** Removing vs. Adding On	Open Number Lines & Equations
NOVEMBER	
No Problem Strings	N/A
DECEMBER	
MULTIPLYING WITH THE DISTRIBUTIVE PROPERTY **String 7** 4 × 7, 3 × 7, 5 × 7, 7 × 7, 9 × 7, 7 × 9, 12 × 7 **String 8** 4 × 6, 5 × 6, 10 × 6, 9 × 6, 12 × 6, 15 × 6, 12 × 8	Number Lines, Open Arrays, Ratio Tables, Equations
JANUARY–MAY	
No Problem Strings	N/A

Problem Strings in Bridges 2nd Edition Grade 4

LOCATION	TITLE	MODELS
UNIT 1		
Module 1, Session 2	**Number Lines** Doubling: $4 \times 2 = p$, $8 \times 2 = p$, $12 \times 2 = p$, $16 \times 2 = p$, $24 \times 2 = p$, $32 \times 2 = p$	Open Number Line, Equations
Module 1, Session 3	**Ratio Table** Doubling one factor while the other stays the same: $4 \times 5 = f$, $8 \times 5 = f$, $16 \times 5 = f$, $32 \times 5 = f$	Ratio Tables
Module 2, Session 4	**One More or One Less** 3 × 4, 3 × 8, 3 × 9, 3 × 7, 4 × 7, 4 × 8, 4 × 9	Open Arrays, Equations
UNIT 2		
Module 3, Session 3	**Doubling & Halving** 4 × 6, 8 × 6, 8 × 3, 8 × 6, 4 × 12, 2 × 24, 1 × 48; $3 \times 14 = 6 \times 7$; $5 \times 12 \neq 10 \times 24$	Open Arrays, Equations
UNIT 3		
	No Problem Strings	N/A
UNIT 4		
Module 1, Session 4	**Give & Take** 36 + 99, 697 + 178, 3999 + 1747, and 3 other combinations	Open Number Lines, Vertical Equations
UNIT 5		
	No Problem Strings	N/A
UNIT 6		
Module 1, Session 3	**Packs of Pens:** 14 pens in each pack, how many pens in x packs? 1 × 14, 2 × 14, 10 × 14, 20 × 14, 3 × 14, 5 × 14, 15 × 14, 23 × 14	Ratio Tables, Open Arrays
Module 1, Session 4	**An Over Strategy** 26 bottles of water in each case, how many bottles in x cases? 2 × 26, 4 × 26, 8 × 26, 10 × 26, 9 × 26, 100 × 26, 99 × 26	Ratio Tables, Equations
Module 1, Session 7	**Doubling & Halving** 24 × 25, 12 × 50, 6 × 100; 13 × 8, 26 × 4, 52 × 2, 104 × 1; 32 × 21	Open Arrays
Module 3, Session 4	**Division, Fractions & Decimals** In the context of kids sharing money: \$5.00 ÷ 5, \$5.00 ÷ 10, \$4.00 ÷ 10, \$8.00 ÷ 10, \$2.00 ÷ 10, \$3.00 ÷ 10, \$1.00 ÷ 10	Equations, Ratio Tables
Module 4, Session 1	**Equivalent Ratios** In the context of people sharing money: \$20 ÷ 10, \$40 ÷ 10, \$20 ÷ 5, \$20 ÷ 20, \$10 ÷ 10, \$5 ÷ 10, \$2.50 ÷ 5, \$2.50 ÷ 10	Equations
UNITS 7–8		
	No Problem Strings	N/A

Problem Strings in Number Corner 2nd Edition Grade 4

TITLE	MODELS
SEPTEMBER: MULTIPLICATION MODELS	
String 1 Double-Half: 5 × 8, 10 × 8, 10 × 16, 5 × 16, 15 × 16, 30 × 8, 4 × 60 **String 2** Double-Half, Associative Property: 8 × 10, 4 × 20, 2 × 40, 16 × 5, 32 × 5, 16 × 10 **String 3** Partial Products, Over Strategy: 3 × 6, 3 × 60, 3 × 61, 3 × 59, 4 × 59, 4 × 39	Closed Arrays Number Line Equations Open Arrays
OCTOBER: RATIO TABLES IN CONTEXT	
String 4 Sticks of Clay: 2 × 4, 4 × 4, 8 × 4, 7 × 4, 9 × 4, 14 × 4, 18 × 4) **String 5** Dozens of Eggs, Distributive Property: 1 × 12, 3 × 12, 6 × 12, 7 × 12, 12 × 12, 11 × 12, 10 × 12, 13 × 12 **String 6** Park Passes for $7 each: 1 × 7, 2 × 7, 4 × 7, 8 × 7, 12 × 7, 20 × 7, 19 × 7, 16 × 7, 17 × 7	Ratio Tables Equations
NOVEMBER: MULTI-DIGIT ADDITION STRATEGIES	
String 7 Get to a Friendly Number **String 8** Give & Take **String 9** Place Value, Associative Property, Decomposing, Standard Algorithm	Open Number Lines Equations
DECEMBER: MULTI-DIGIT SUBTRACTION STRATEGIES	
String 10 Removal (Take-Away) and Differencing **String 11** Constant Difference **String 12** Looking at the Standard Algorithm with Money: Example: I have 14 dimes and 6 pennies. If I give away 19 pennies, how many cents do I have left?	Open Number Lines Equations
JANUARY: DIVISION STRATEGIES (WITH AND WITHOUT REMAINDERS)	
String 13 Jumps on the Open Number Line, Inverse Relationship Between Multiplication & Division: 10 × 9, 90 ÷ 9, 90 ÷ 10, 3 × 30, 90 ÷ 30, 90 ÷ 3, 6 × 15, 90 ÷ 6, 90 ÷ 15, 92 ÷ 30, 92 ÷ 6 **String 14** Area, Inverse Relationship between Multiplication & Division, Multiplying in Chunks to Build up to the Quotient: 14 × 10, 140 ÷ 10, 140 ÷ 14, 70 ÷ 14, 14 × 15, 210 ÷ 14, 224 ÷ 14 **String 15** Marking Pens in Packs of 8—Inverse Relationship between Multiplication & Division, Multiplying in Chunks to Build up to the Quotient, Partial Products: 10 × 8, 80 ÷ 8, 6 × 8, 128 ÷ 8, 136 ÷ 8, 160 ÷ 8, 168 ÷ 8, 184 ÷ 8, 84 ÷ 6, 87 ÷ 6, 95 ÷ 6	Open Number Lines Open Arrays Ratio Tables Equations
FEBRUARY: ADDING & SUBTRACTING FRACTIONS WITH LIKE & UNLIKE DENOMINATORS	
String 16, Pt. 1 Decomposing Fractions: $\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$ or $2 \times \frac{1}{3}$; $\frac{4}{3} = 4 \times \frac{1}{3}$, $1 + \frac{1}{3}$, $2 \times \frac{2}{3}$, etc. **String 16, Pt. 2** Adding & Subtracting Fractions with Like Denominators: $\frac{4}{3} + \frac{2}{3}$, $\frac{6}{3} - \frac{2}{3}$, etc. **String 17** Adding Fractions with Unlike Denominators in the Context of Money: $\frac{1}{2} + \frac{1}{4}$ is like adding half a dollar and a quarter. That's 75¢ or $0.75 or $\frac{75}{100} = \frac{15}{20} = \frac{3}{4}$ **String 18** Adding & Subtracting Fractions with Unlike Denominators in the Context of Time: $\frac{1}{2} + \frac{1}{4}$ is like adding half an hour and a quarter of an hour. That's 45 minutes or $\frac{45}{60} = \frac{9}{12} = \frac{3}{4}$	Bar Models Coins Minutes & Hours Clock Faces Equations
MARCH: GENERATING EQUIVALENT FRACTIONS	
String 19 Equivalent Fractions on a Clock: What fraction of an hour are 30, 15, 45, 20, 40, and 10 minutes? **String 20** More Equivalent Fractions on a Clock: How many equivalent fractions can you find for $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{1}{6}$ on a clock face? **String 21** Using a Hundreds Grid to Determine If Two Fractions Are Equivalent: $\frac{1}{4} = \frac{25}{100}$ T or F? $\frac{3}{5} = \frac{6}{10}$ T or F? $\frac{1}{3} = \frac{3}{10}$ T or F?	Minutes & Hours Clock Faces Blank 10 × 10 Grids
APRIL: MORE DIVISION STRATEGIES	
String 22 Halving or Doubling the Divisor While the Dividend Remains Constant: 48 ÷ 8, 48 ÷ 4, 48 ÷ 2; 36 ÷ 2, 36 ÷ 4, 36 ÷ 8; 24 ÷ 6, 48 ÷ 6, 96 ÷ 6; 72 ÷ 8, 36 ÷ 8, 18 ÷ 8 **String 23** Doubling or Halving Both the Dividend and the Divisor / Doubling the Dividend and Halving the Divisor / Halving the Dividend and Doubling the Divisor: 36 ÷ 4, 72 ÷ 4, 72 ÷ 8; 48 ÷ 6, 24 ÷ 6, 24 ÷ 3; 40 ÷ 8, 80 ÷ 8, 80 ÷ 4; 60 ÷ 6, 30 ÷ 6, 30 ÷ 12 **String 24** Multiplying to Divide: the Relationship Between Multiplication & Division, Over Strategy: 12 × 10, 12 × 9; 120 ÷ 12, 108 ÷ 12, 135 ÷ 15; 140 ÷ 14, 126 ÷ 14, 154 ÷ 14	Open Arrays Ratio Tables Equations
MAY: MULTIPLYING FRACTIONS & WHOLE NUMBERS	
String 25, Pt. 1 Doubling One Factor While the Other Remains Constant: 2 × 24, 4 × 24, 8 × 24 **String 25, Pts. 2 & 3** Multiplying a Whole Number by a Unit Fraction: $\frac{1}{2} \times 24$, $\frac{1}{4} \times 24$, $\frac{1}{8} \times 24$, $\frac{1}{3} \times 24$, $\frac{1}{6} \times 24$ **String 26** Multiplying Whole Numbers by Unit and Non-Unit Fractions: $\frac{1}{4} \times 16$, $\frac{1}{8} \times 16$, $\frac{2}{8} \times 16$, $\frac{3}{8} \times 16$, $\frac{5}{8} \times 16$; $\frac{3}{4} \times 24$ **String 26** Multiplying Whole Numbers by Unit and Non-Unit Fractions Using Ratio Tables: $\frac{1}{6} \times 36$, $\frac{2}{6} \times 36$, $\frac{4}{6} \times 36$, $\frac{5}{6} \times 36$; $\frac{1}{8} \times 32$, $\frac{2}{8} \times 32$, $\frac{3}{8} \times 32$, $\frac{5}{8} \times 32$, $\frac{6}{8} \times 32$	Tile Arrays Ratio Tables Equations

Problem Strings in Bridges 2nd Edition Grade 5

LOCATION	TITLE	MODELS
UNIT 1		
Module 1, Session 5	**Double a Dimension, Double the Area** 4 × 3, 4 × 6, 8 × 6, 4 × 12, 2 × 24, 1 × 48, 3 × 16	Closed Arrays, Doubling/Halving
Module 2, Session 1	**Doubling & Halving** 3 × 4, 3 × 8, 6 × 8, 6 × 4, 8 × 9, 4 × 18, 2 × 36, 8 × 7	Closed Arrays, Doubling/Halving
Module 2, Session 3	**Ten Times, More or Less** 3 × 6, 3 × 60, 3 × 61, 3 × 59	Closed & Open Arrays
Module 2, Session 6	**Multiplication** Distributive property: 17 × 11 = (8 × 11) + (9 × 11)	Open Arrays, Equations
Module 3, Session 1	**An Over Strategy** 10 × 27, 9 × 27, 100 × 27, 99 × 77	Ratio Tables, Equations
Module 3, Session 2	**Partial Products** 3 × 23, 30 × 23, 33 × 23, 60 × 23, 66 × 23, 43 × 32	Open Arrays, Partial Products, Equations
Module 3, Session 3	**Five Is Half of Ten** 10 × 26, 5 × 26, 15 × 26; 10 × 78, 5 × 78	Ratio Tables, Open Arrays, Equations
Module 3, Session 4	**Another Over Strategy** 21 × 60, 21 × 59	Open Arrays, Equations
UNIT 2		
Module 1, Session 1	**Using Money** To add fractions with unlike denominators: $\frac{1}{2} + \frac{1}{4}$ is like adding half a dollar and a quarter. That's 75¢ or \$0.75 or $\frac{75}{100} = \frac{15}{20} = \frac{3}{4}$	Money Value Pieces, Equations
Module 1, Session 2	**Money Fractions** Adding & subtracting fractions with unlike denominators: $\frac{1}{2} - \frac{1}{5}$ is like \$0.50 – \$0.20; that's \$0.30 = $\frac{30}{100} = \frac{3}{10}$	Money Value Pieces, Equations
Module 1, Session 3	**More Fraction Models** Using money to help add & subtract fractions with unlike denominators.	Money Value Pieces, Equations, Open Number Lines
Module 1, Session 4	**Clock Fractions** Adding fractions with unlike denominators: $\frac{1}{2} + \frac{1}{3}$ is like adding 30 minutes and 20 minutes. That's 50 minutes or $\frac{50}{60} = \frac{5}{6}$	Clock Faces, Equations
Module 1, Session 5	**Fraction Subtraction** Using money and time—fractions of a dollar and fractions of an hour—to help subtract fractions with unlike denominators.	Clock Faces, Money Value Pieces, Equations
Module 2, Session 1	**Multiplying Whole Numbers by Fractions** $\frac{1}{3}$ of 24, $\frac{1}{3} \times 24$	Tile Arrays, Equations
Module 2, Session 6	**Fraction Model Practice** Using money, time, and the double number line to help add & subtract fractions with unlike denominators.	Clock Faces, Money Value Pieces, Double Number Line
UNIT 3		
Module 1, Session 1	**Give & Take** Adding whole numbers & decimals: \$1.99 + \$1.57 = \$2.00 + \$1.56 = \$3.56	Open Number Line, Equations, Vertical Combinations
Module 1, Session 2	**Another Give & Take String** Adding whole numbers & decimals: \$75.87 + 74.89 = \$75.76 + \$75.00 = \$150.76	Open Number Line, Equations, Vertical Combinations
Module 2, Session 1	**Decimal Subtraction** Removal vs. differencing: when the minuend and the subtrahend are close, it's more efficient to find the difference between them than to use a take away strategy. When they're far apart, it's more efficient to use a take away or removal strategy. 34.3 – 0.4 is more easily solved using removal, while 31.3 – 30.8 is more easily solved by finding the difference between the two.	Open Number Line, Equations
Module 2, Session 6	**Constant Difference** Whole numbers & decimals: 17.2 – 8.9 = 17.3 – 9.0 if you add 0.1 to each, and 17.3 – 9.0 is much more easily solved.	Open Number Line, Equations
Module 2, Session 7	**More Constant Difference** Whole numbers & decimals: 6.1 – 3.96 = 6.14 – 4.00 if you add 0.04 to each, and 6.14 – 4.00 is more easily solved.	Open Number Line, Equations

Problem Strings in Bridges 2nd Edition Grade 5 (CONTINUED)

LOCATION	TITLE	MODELS
UNIT 4		
Module 1, Session 1	**Boxes & Pencils** If 1 box has 35 pencils, how many pencils are in 4 boxes, 40 boxes, 39 boxes, and so on? If I have 1,400 pencils, how many boxes is that? 4 × 35, 40 × 35, 39 × 35, 44 × 35, 140 ÷ 35, 1400 ÷ 35, 1435 ÷ 35, etc.	Ratio Tables, Equations
Module 1, Session 2	**Half-Tens Facts** 10 × 18, 5 × 18, 15 × 18, 10 × 180, 5 × 180, 15 × 180	Ratio Tables, Equations
Module 1, Session 3	**Doubling & Halving** 24 × 4 = 48 × 2 = 96 × 1; 24 × 25 = 12 × 50 = 6 × 100	Open Arrays, Equations
Module 2, Session 1	**Multiplying Fractions, Decimals & Whole Numbers** 25 × 32, $\frac{1}{4}$ of 32, 0.25 × 32	Open Arrays, Ratio Tables, Equations
Module 2, Session 2	**Multiplication Relationships** $\frac{1}{4}$ × 8, $\frac{3}{4}$ of 8, 0.75 × 8, 75 × 9	Ratio Tables, Equations
Module 1, Session 3	**Fractions as Operators** 0.25 × 8, 26 × 8, 0.26 × 8, 25 × 84	Ratio Tables, Equations
UNIT 5		
Module 1, Session 5	**Trading Places** 8 × $\frac{3}{4}$, 3 × $\frac{8}{4}$, $\frac{3}{5}$ × 20, $\frac{20}{5}$ × 3	Ratio Tables, Equations
UNIT 6		
Module 4, Session 1	**Doubling & Halving with Banners** 2 $\frac{1}{2}$ × 18, 1 $\frac{1}{4}$ × 36; 5 × 9, 4.5 × 10,	Open Arrays, Equations
UNIT 7		
Module 1, Session 2	**Partial Quotients** 36 ÷ 6, 360 ÷ 6, 30 ÷ 6, 390 ÷ 6, 330 ÷ 6	Open Arrays, Equations
Module 1, Session 3	**Partial Products & Partial Quotients** Using known multiplication combinations to help divide; pulling dividends apart into chunks that are easier to divide: 280 ÷ 14 can be solved using the fact that 10 × 14 = 140, and 20 × 14 = 280.	Ratio Tables, Equations
Module 1, Session 5	**Equivalent Ratios** Applied to division of whole numbers by other whole numbers, but also whole numbers by unit fractions, e.g., 5 ÷ $\frac{1}{4}$ = 10 ÷ $\frac{1}{2}$ = 20 ÷ 1.	Ratio Tables
Module 1, Session 6	**Minutes Per Mile** Equivalent ratios in context, e.g., If a student walked $\frac{1}{4}$ of a mile in 6 minutes, how fast was she walking? What was her pace per mile? 6 ÷ $\frac{1}{4}$ = 12 ÷ $\frac{1}{2}$ = 24 ÷ 1 = 24 minutes per mile	Ratio Tables
Module 2, Session 1	**Fraction Division with Money** Dividing whole numbers by unit fractions in the context of money, e.g., How many half-dollars are there in $3.00? 3 ÷ $\frac{1}{2}$ = 6 ÷ 1 = 6	Ratio Tables
Module 2, Session 3	**Fraction Division on a Clock** Dividing whole numbers by unit fractions and vice versa in the context of fractions of an hour, e.g., How many half hours are there in 2 hours? 2 ÷ $\frac{1}{2}$ = 4; What is $\frac{1}{4}$ of an hour divided into three equal amounts of time? $\frac{1}{4}$ ÷ 3 = $\frac{1}{12}$	Clock Faces, Equations
Module 2, Session 4	**Overs & Unders** Partial quotients: 726 ÷ 66 = (660 + 66) ÷ 6 = (660 ÷ 6) + (66 ÷ 6) = 10 + 1 = 11	Ratio Tables, Equations
Module 2, Session 5	**Over & Under with Division** Partial quotients: 702 ÷ 78 = (780 – 78) ÷ 78 = (780 ÷ 78) – (78 ÷ 78) = 10 – 1 = 9	Ratio Tables, Equations
Module 3, Session 1	**Powers of Ten** Multiplying & dividing by powers of ten: 4.5 × 103 = 4.5 × (10 × 10 × 10) = 4.5 × 1,000 = 4,500	Equations
UNIT 8		
	No Problem Strings	N/A

Problem Strings in Number Corner 2nd Edition Grade 5

TITLE	MODELS
SEPTEMBER: ADDITION & SUBTRACTION STRATEGIES (WHOLE NUMBERS & DECIMALS)	
String 1 Give & Take **String 2** Removal vs. Differencing **String 3** Constant Difference	Open Number Line Equations
OCTOBER: FRACTION ADDITION WITH MONEY & CLOCK MODELS	
String 4 Addition of fractions that correspond to coin values, leading to common denominators of 100: $\frac{1}{4} + \frac{1}{2}$, $\frac{1}{4} + \frac{1}{10}$, etc. **String 5** Addition of fractions that correspond to fractions on a clock face, leading to common denominators of 60: $\frac{1}{4} + \frac{1}{3}$, $\frac{1}{2} + \frac{1}{3}$, $\frac{1}{4} + \frac{1}{12}$, etc.	Equations Clock Faces Money Value Pieces
NOVEMBER: FRACTION SUBTRACTION WITH MONEY & CLOCK MODELS	
String 6 Subtraction of fractions that correspond to fractions on a clock face, leading to common denominators of 60: $\frac{1}{2} - \frac{1}{3}$, $\frac{1}{3} - \frac{1}{4}$, $\frac{1}{4} - \frac{1}{6}$, $\frac{1}{6} - \frac{1}{12}$, etc. **String 7** Subtraction of fractions that correspond to coin values, leading to common denominators of 100: $\frac{9}{10} - \frac{1}{2}$, $\frac{7}{10} - \frac{1}{4}$, $\frac{1}{2} - \frac{1}{20}$, $\frac{7}{10} - \frac{1}{4}$, etc. **Strings 8 & 9** Subtraction of fractions that lend themselves best to the clock model, best to the money model, or could be solved equally well using either model: $\frac{2}{3} - \frac{1}{12}$, $\frac{7}{10} - \frac{1}{4}$, $\frac{5}{6} - \frac{3}{4}$. Students are asked to discuss the model(s) that would work best for each problem in the string.	Clock Faces Equations Money Value Pieces
DECEMBER: MULTIPLICATION & DIVISION (WHOLE NUMBERS & DECIMALS)	
String 10 Whole Numbers—Doubling, Partial Products, Over/Under, Relationship between Multiplication & Division: 2×22, 4×22, 8×22, 10×22, 9×22, 12×22, 100×22, 99×22, $110 \div 22$ **String 11** Decimals—Doubling, Partial Products, Over/Under, Relationship between Multiplication & Division: 2×1.2, 4×1.2, 8×1.2, 10×1.2, 9×1.2, 100×1.2, 101×1.2, 99×1.2, $13.2 \div 1.2$ **String 12** Decimals—Doubles, Halves, Over/Under, Partial Products: 4×1.75, 8×1.75, 10×1.75, 9×1.75, 5×1.75, 15×1.75, 99×1.75, $171.50 \div 1.75$, $346.50 \div 1.75$	Word Problems Equations Ratio Tables Money
JANUARY: MORE MULTIPLICATION & DIVISION STRINGS	
String 13 Multiplying Decimals by Whole Numbers; Dividing Whole Numbers by Decimals: 2 × \$1.50, 20 × \$1.50, 4 × \$1.50, 24 × \$1.50, 42 × \$1.50, \$180 ÷ \$1.50 **String 14** Multiplying Decimals by Whole Numbers; Dividing Decimals by Decimals: 10 × \$1.25, 9 × \$1.25, 5 × \$1.25, 100 × \$1.25, 50 × \$1.25, 49 × \$1.25, \$123.75 ÷ \$1.25 **String 15** Multiplying Decimals by Whole Numbers; Dividing Decimals by Decimals: 9 × \$1.20, 5 × \$1.20, 15 × \$1.20, 14 × \$1.20, 100 × \$1.20, 50 × \$1.20, 25 × \$1.20, 75 × \$1.20, \$91.20 ÷ \$1.20	Ratio Tables
FEBRUARY: MULTIPLYING WHOLE NUMBERS BY FRACTIONS	
String 16 $\frac{1}{2} \times 16$, $\frac{1}{4} \times 16$, $\frac{3}{4} \times 16$, $\frac{1}{8} \times 16$, $\frac{2}{8} \times 16$, $\frac{3}{8} \times 16$, and so on **String 17** $\frac{1}{2} \times 20$, $\frac{1}{4} \times 20$, $\frac{3}{4} \times 20$, $\frac{1}{5} \times 20$, $\frac{2}{5} \times 20$, and so on **String 18** $\frac{1}{2} \times 36$, $\frac{1}{4} \times 36$, $\frac{1}{8} \times 36$, $\frac{3}{4} \times 36$, and so on **String 19** Multiplying a whole number by a fraction is the same as dividing that whole number by the reciprocal of the fraction, e.g., $\frac{1}{2} \times 48$ is the same as $48 \div 2$; $\frac{1}{3} \times 48$ is the same as $48 \div 3$.	Arrays Equations
MARCH: FRACTION ADDITION & SUBTRACTION	
String 20 Adding unit fractions, improper fractions, and mixed numbers with unlike denominators by rewriting the numbers so they have common denominators, e.g., $\frac{1}{4} + \frac{1}{5} = \frac{5}{20} + \frac{4}{20} = \frac{9}{20}$ **String 21** Adding and subtracting unit fractions, improper fractions, and mixed numbers with unlike denominators by rewriting the numbers so they have common denominators, e.g., $3\frac{1}{3} - 2\frac{1}{6} = 3\frac{2}{6} - 2\frac{1}{6} = 1\frac{1}{6}$ **String 22** Subtracting fractions, improper fractions, and mixed numbers with unlike denominators by rewriting the numbers so they have common denominators, e.g., $\frac{3}{4} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12} = \frac{1}{12}$ **String 23** Adding & subtracting fractions & mixed numbers with unlike denominators by rewriting the numbers so they have common denominators	Equations
APRIL: FRACTION MULTIPLICATION & DIVISION	
String 24 Multiplying one unit fraction by another unit fraction: $\frac{1}{4} \times \frac{1}{5}$, $\frac{1}{2} \times \frac{1}{3}$, $\frac{1}{4} \times \frac{1}{3}$; moving toward the generalization that we multiply numerator times numerator and denominator times denominator in finding the product of two fractions **String 25** Multiplying non-unit fractions by unit and non-unit fractions ($\frac{1}{4} \times \frac{1}{5}$, $\frac{3}{4} \times \frac{1}{5}$, $\frac{3}{4} \times \frac{2}{5}$) **String 26** Dividing whole numbers by unit fractions in the context of word problems: I had 4 candy bars. I gave each of my friends $\frac{1}{5}$ of a candy bar. How many friends did I give a piece of the candy bar to? $4 \div \frac{1}{5} = 20$ friends **String 27** Dividing unit fractions by whole numbers in the context of word problems: I have $\frac{1}{2}$ of a candy bar. I will share it with 2 friends so there are 3 sharing the $\frac{1}{2}$ bar. How much will we each get? $\frac{1}{2} \div 3 = \frac{1}{6}$ of a candy bar each	Rectangular Arrays Equations Bar Models Ratio Tables
MAY: FRACTION MULTIPLICATION & DIVISION	
String 28 Multiplying unit by unit and non-unit by non-unit fractions **String 29** Multiplying non-unit by non-unit fractions **String 30** Dividing whole numbers by unit fractions in the context of word problems **String 31** Dividing unit fractions by whole numbers in the context of word problems	Rectangular Arrays Equations Bar Models